CSIR NET/JRF

Mathematical Science

CSIR NET 2023 (07-06-2023)

PART-A

(Mathematical Sciences)

(1.) On a spherical globe of radius 10 units, the distance between A and B is 25 units. If it is uniformly expanded to a globe of radius 50 units, the distance between them in the same units would be

(a.) 75

(b.) 125

(c.) 150

(d.) 625

(2.) The ratio of ages of a mother and daughter is 14:1 at present. After your years, the ratio of their ages will be 16:3. What was the age of mother when the daughter was born?

(a.) 26

(b.) 28

(c.) 30

(d.) 32

(3.) In an examination containing 10 questions, each correct answer is awarded 2 marks, each incorrect answer is awarded –1 and each unattempted question is awarded zero. What of the following CANNOT be a possible score in the examination?

(a.) –9

(b.) –7

(c.) 17

(d.) 19

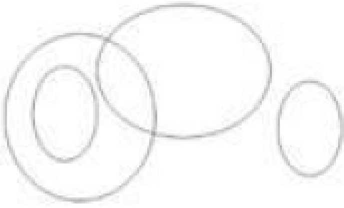
(4.) If the speed of a train is increased by 20% , its travel time between two stations reduces by 2 hrs. If its speed is decreased by 20%, the travel time increases by 3 hrs. What is the normal duration of travel (in hrs)?

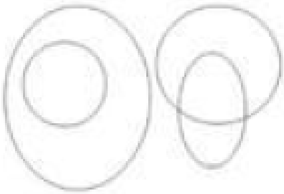
(a.) 11.5

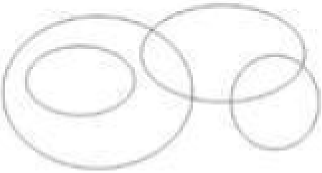
(b.) 12.0

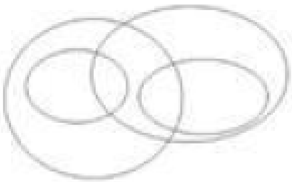
(c.) 13.2

(d.) 14.0

(5.) An approximate diagram to depict the relationships between the categories INSECTS, BIRDS, EXTINCT ANIMALS and PEACOCKS is (a.) 

(b.) 

(c.) 

(d.) 

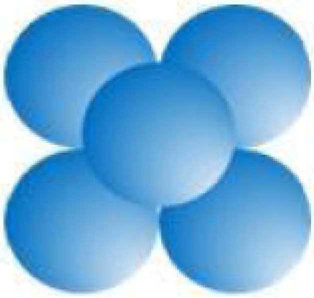
(6.) In a meeting of 45 people, there are 40 people who know one another and the remaining know no one. People who knew each other only hug, whereas those who do not know each other only shake hands. How many handshakes occur in this meeting?

(a.) 225
(b.) 10
(c.) 210
(d.) 200

(7.) The standard deviation of data $x_1, x_2, x_3, \ldots, x_n$ is $\sigma(\sigma > 0)$. Then the standard deviation of data $3x_1 + 2$, $3x_2 + 2$, $3x_3 + 2, \ldots, 3x_n + 2$ is

(a.) 3σ
(b.) σ
(c.) $3\sigma + 2$
(d.) 9σ

(8.) Five identical incompressible spheres of radius 1 unit are stacked in a pyramidal form as shown in the figure. The height of the structure is



Top view

(a.) $2+\sqrt{2}$

(b.) $2+\sqrt{3}$

(c.) $2+2\sqrt{\frac{2}{3}}$

(d.) 3

(9.) A person takes loan of Rs. 1,50,000 at a compound interest rate of 10% per annum. If the loan is repaid at the end of the 3rd year, what is the total interest paid?

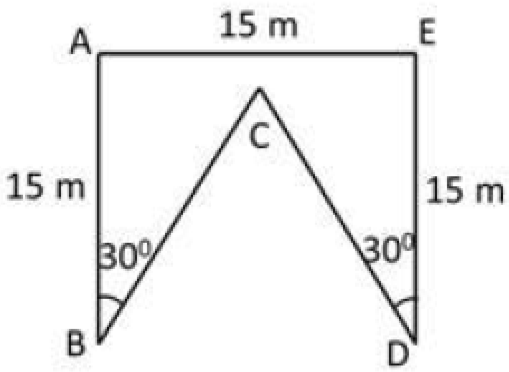
(a.) 45000

(b.) 82600

(c.) 94600

(d.) 49650

(10.) The figure shows map of a field bounded by ABCDE. If AB and DE are perpendicular to AE, then the perimeter of the field is



(a.) 70 m

(b.) 75 m

(c.) 80 m

(d.) 85 m

(11.) A boy can escape through a window of size at least 4 feet. The 28 windows of a house are of sizes 2,3,4 or 5 feet and their numbers are proportional to their sizes. The number of windows available for the boy to escape through is

(a.) 2

(b.) 9

(c.) 10

(d.) 18

(12.) In a group of 7 people, 4 have exactly one sibling and 3 have exactly two siblings. Two people selected at random from the group, what is the probability that they are NOT sibling?

(a.) 5/21

(b.) 16/21

(c.) 3/7

(d.) 4/7

(13.) Peron A tells the truth 30% of the times and B tells the truth 40% of the times, independently. What is the minimum probability that they would contradict each other?

(a.) 0.18

(b.) 0.42

(c.) 0.46

(d.) 0.50

(14.) Two datasets A and B have the same mean. Which of the following MUST be true?

(a.) Sum of the observations in A = Sum of the observations in B

(b.) Mean of the squares of the observations in A = Mean of the squares of the observations in B

(c.) If the two datasets are combined, then the mean of the combined dataset = mean of A + mean of B

(d.) If the two datasets are combined, then the mean of the combined dataset = mean of A

(15.) In an assembly selection, parties A,B,C,D and E won 30, 25, 20, 10 and 4 seats, respectively; whereas independents won 9 sets. Based on this data, which of the following statements must be INCORRECT?

(a.) No party has majority

(b.) A and C together can from the government

(c.) A and D with the support of independents get the majority

(d.) An MLA from E can become Chief Minister

(16.) The difference of the squares of two distinct two-digit numbers with one being obtained by reversing the digits of the other in always divisible by

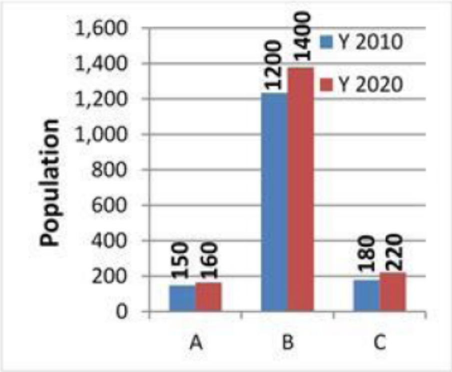
(a.) 4

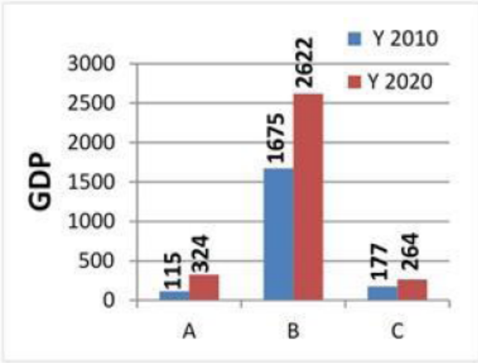
(b.) 6

(c.) 10

(d.) 11

(17.) The populations and gross domestic product (GDP) in billion USD of three countries A, B and C in the years 2010 and 2020 are shown in the two figures below:





In terms of increases in per capita GDP from 2010-2020, their ranking from high to low is

(a.) A,B,C

(b.) B,A,C

(c.) B,C,A

(d.) C,A,B

(18.) A device needs 4 batteries to run. Each battery runs for 2 days. If there are a total of 6 batteries available, what is the maximum number of days for which the device can be run by strategically replacing the batteries till all the batteries are completely drained of power?

(a.) 2

(b.) 3

(c.) 4

(d.) 5

(19.) A and B have in their collection, coins of Rs. 1, Rs. 2, Rs. 5 and Rs. 10 in the ratio 3:2:2:1 and 4:3:2:1, respectively. The total number of coins with each of them is equal. If the value of coins with A is Rs. 270/-, what is the value of the coins (in Rs) with B?

(a.) 213

(b.) 240

(c.) 275

(d.) 282

(20.) Consider the following paragraph:

THE ABILITY OF REASON ACCURATELY IS VERY IMPORTANT, AS IS THE ABILITY TO COUNT. AS AN EXERCISE IN BOTH, LET US COUNT HOW MANY TIMES THE LETTER "E" OCCURS IN THIS PARAGRAPH. THE CORRECT COUNT IS _____.

Which option when put in the blank in the above paragraph will make the final sentence accurate?

(a.) Sixteen

(b.) SEVENTEEN

(c.) EIGHTEEN

(d.) NINETEEN

PART-B

(Mathematical Sciences)

(21.) Consider the function f defined by $f(z)=\dfrac{1}{1-z-z^2}$ for $z\in\mathbb{C}$ such that $1-z-z^2\neq 0$. Which of the following statements is true?

(a.) f is an entire function

(b.) f has a simple pole at $z=0$

(c.) f has a Taylor series expansion $f(x)=\sum_{n=0}^{\infty}a_n z^n$, where coefficients a_n are recursively defined as follows: $a_0=1, a_1=0$ and $a_{n+2}=a_n+a_{n+1}$ for $n\geq 0$

(d.) f has a Taylor series expansion $f(z)=\sum_{n=0}^{\infty}a_n z^n$, where coefficients a_n are recursively defined as follows: $a_0=1, a_1=1$ and $a_{n+2}=a_n+a_{n+1}$ for $n\geq 0$

(22.) Let $X_1,\ldots,X_7$ and $Y_1,\ldots,Y_7$ be two random samples drawn independently from two populations with continuous CDFs F and G respectively. Consider the Wald-Wolfowitz run test in the context of the following two sample testing problems: $H_0: F(x)=G(x)\,\forall x$ vs. $H_1: F(x)\neq G(x)$ for some x. If the random variable R denotes the total number of runs in the combined ordered arrangement of the two given samples, then which of the following is true?

(a.) $P_{H_0}(R=6)=\dfrac{28}{286}$, $P_{H_0}(R=9)=\dfrac{28}{143}$

(b.) $P_{H_0}(R=6)=\dfrac{21}{286}$, $P_{H_0}(R=9)=\dfrac{15}{286}$

(c.) $P_{H_0}(R=6)=\dfrac{21}{286}$, $P_{H_0}(R=9)=\dfrac{28}{143}$

(d.) $P_{H_0}(R=6)=\dfrac{21}{286}$, $P_{H_0}(R=9)=\dfrac{15}{286}$

(23.) For the unknown $y:[0,1]\to\mathbb{R}$, consider the following two-point boundary value problem:

$$\begin{cases} y''(x)+2y(x)=0 \text{ for } x\in(0,1), \\ y(0)=y(1)=0 \end{cases}$$

It is given that the above boundary value problem corresponds to the following integral equation:

$$y(x)=2\int_0^1 K(x,t)\,y(t)\,dt \text{ for } x\in[0,1].$$

Which of the following is the kernel $K(x,t)$?

(a.) $K(x,t)=\begin{cases} t(1-x) & \text{for } t<x \\ x(1-t) & \text{for } t>x \end{cases}$

(b.) $K(x,t)=\begin{cases} t^2(1-x) & \text{for } t<x \\ x^2(1-t) & \text{for } t>x \end{cases}$

(c.) $K(x,t) = \begin{cases} \sqrt{t}(1-x) & \text{for } t < x \\ \sqrt{x}(1-t) & \text{for } t > x \end{cases}$

(d.) $K(x,t) = \begin{cases} \sqrt{t^3}(1-x) & \text{for } t < x \\ \sqrt{x^3}(1-t) & \text{for } t > x \end{cases}$

(24.) Let A be a 3×3 real matrix whose characteristic polynomial $p(T)$ is divisible by T^2. Which of the following statements is true?

(a.) The eigenspace of A for the eigenvalue 0 is two-dimensional

(b.) All the eigenvalues of A are real

(c.) $A^3 = 0$

(d.) A is diagonalizable

(25.) Which of the following is a valid cumulative distribution function?

(a.) $F(x) = \begin{cases} \dfrac{1}{2+x^2} & \text{if } x < 0, \\ \dfrac{2+x^2}{3+x^2} & \text{if } x \geq 0 \end{cases}$

(b.) $F(x) = \begin{cases} \dfrac{1}{2+x^2} & \text{if } x < 0, \\ \dfrac{2+x^2}{3+2x^2} & \text{if } x \geq 0 \end{cases}$

(c.) $F(x) = \begin{cases} \dfrac{1}{2+x^2} & \text{if } x < 0, \\ \dfrac{2\cos(x)+x^2}{4+x^2} & \text{if } x \geq 0 \end{cases}$

(d.) $F(x) = \begin{cases} \dfrac{1}{2+x^2} & \text{if } x < 0, \\ \dfrac{1+x^2}{4+x^2} & \text{if } x \geq 0 \end{cases}$

(26.) Let $f:\mathbb{R}^2 \to \mathbb{R}$ be a locally Lipschitz function. Consider the initial value problem $\dot{x} = f(t,x),\ x(t_0) = x_0$ for $(t_0, x_0) \in \mathbb{R}^2$. Suppose that $J(t_0, x_0)$ represents the maximal interval of existence for the initial value problem. Which of the following statements is true?

(a.) $J(t_0, x_0) = \mathbb{R}$

(b.) $J(t_0, x_0)$ is an open set

(c.) $J(t_0, x_0)$ is a closed set

(d.) $J(t_0, x_0)$ could be an empty set

(27.) Let $x, y \in [0,1]$ be such that $x \neq y$. Which of the following statements is true for every $\varepsilon > 0$?

(a.) There exists a positive integer N such that $|x-y| < 2^n \varepsilon$ for every integer $n \geq N$

(b.) There exists a positive integer N such that $2^n \varepsilon < |x-y|$ for every integer $n \geq N$

(c.) There exists a positive integer N such that $|x-y| < 2^{-n}\varepsilon$ for every integer $n \geq N$

(d.) For every positive integer N, $|x-y| < 2^{-n}\varepsilon$ for some integer $n \geq N$

(28.) Consider the series $\sum_{n=1}^{\infty} a_n$, where $a_n = (-1)^{n+1}\left(\sqrt{n+1}-\sqrt{n}\right)$. Which of the following statements is true?

(a.) The series is divergent

(b.) The series is convergent

(c.) The series is conditionally convergent

(d.) The series is absolutely convergent

(29.) Let $l \geq 1$ be a positive integer. What is the dimension of the $\mathbb{R}$-vector space of all polynomials in two variables over $\mathbb{R}$ having a total degree of at most l?

(a.) $l+1$

(b.) $l(l-1)$

(c.) $l(l+1)/2$

(d.) $(l+1)(l+2)/2$

(30.) Let $\{\varepsilon_n : n \geq 1\}$ represents the results of independent rolls of a dice with probability of the face i turning up being $p_i > 0$ for $i = 1,2,\ldots,6$ and $\sum_{i=1}^{6} p_i = 1$. Let $\{X_n : n \geq 0\}$ be the Markov chain on the state space $\{1,2,\ldots,6\}$ where $X_n = \max\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{n+1}\}$. Then, $\lim_{n\to\infty} P(X_n = 4 \mid X_0 = 3)$ equals

(a.) p_4

(b.) 1

(c.) $1-p_3$

(d.) 0

(31.) Consider the quadratic form $Q(x,y,z)$ associated to the matrix $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$. Let

$S = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^2 \mid Q(a,b,c) = 0 \right\}$. Which of the following statements is FALSE?

(a.) The intersection of S with the xy-plane is a line

(b.) The intersection of S with the xz-plane is an ellipse

(c.) S is the union of two planes

(d.) Q is a degenerate quadratic form

(32.) Consider the linear programming problem maximize $x+3y$, subject to $A\begin{pmatrix} x \\ y \end{pmatrix} \le b$, where

$$A = \begin{pmatrix} -1 & -1 \\ 0 & 1 \\ -1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix} \text{ and } b = \begin{pmatrix} -1 \\ 5 \\ 5 \\ 14 \\ 0 \end{pmatrix}$$. Which of the following statements is true?

(a.) The objective function attains its maximum at $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ in the feasible region

(b.) The objective function attains its maximum at $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ in the feasible region

(c.) The objective function attains its maximum at $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in the feasible region

(d.) The objective function does not attain its maximum at $\begin{pmatrix} 14 \\ 0 \end{pmatrix}$ in the feasible region

(33.) If $f(x)$ is a probability density on the real line, then which of the following is NOT a valid probability density?

(a.) $f(x+1)$

(b.) $f(2x)$

(c.) $2f(2x-1)$

(d.) $3x^2 f(x^3)$

(34.) Let $u(x,y)$ be the solution of the Cauchy problem

$uu_x + u_y = 0, x \in \mathbb{R}, y > 0,$

$u(x,0) = x, \ x \in \mathbb{R}.$

Which of the following is the value of $u(2,3)$?

(a.) 2

(b.) 3

(c.) 1/2

(d.) 1/3

(35.) Suppose $x(t)$ is the solution of the following initial value problem in $\mathbb{R}^2$

$\dot{x} = Ax, \ x(0) = x_0$, where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

Which of the following statements is true?

(a.) $x(t)$ is a bounded solution for some $x_0 \neq 0$

(b.) $e^{-6t}|x(t)| \to 0$ as $t \to \infty$, for all $x_0 \neq 0$

(c.) $e^{-t}|x(t)| \to \infty$ as $t \to \infty$, for all $x_0 \neq 0$

(d.) $e^{-10t}|x(t)| \to 0$ as $t \to \infty$, for all $x_0 \neq 0$

(36.) Let f be an entire function that satisfies $|f(z)| \leq e^y$ for all $z = x + iy \in \mathbb{C}$, where $x, y \in \mathbb{R}$. Which of the following statements is true?

(a.) $f(z) = ce^{-iz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$

(b.) $f(z) = ce^{iz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$

(c.) $f(z) = e^{-ciz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$

(d.) $f(z) = e^{ciz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$

(37.) Let X_1, X_2, X_3 and X_4 be independent and identically distributed Bernoulli $\left(\frac{1}{3}\right)$ random variables. Let $X_{(1)}, X_{(2)}, X_{(3)}$ and $X_{(4)}$ denote the corresponding order statistics. Which of the following is true?

(a.) $X_{(1)}$ and $X_{(4)}$ are independent

(b.) Expectation of $X_{(2)}$ is $\frac{1}{2}$

(c.) Variance of $X_{(2)}$ is $\frac{8}{81}$

(d.) $X_{(4)}$ is a degenerate random variable

(38.) Let T be a linear operator on $\mathbb{R}^3$. Let $f(X) \in \mathbb{R}[X]$ denote its characteristic polynomial. Consider the following statements.

A. Suppose T is non-zero and 0 is an eigen value of T. If we write $f(X) = Xg(X)$ in $\mathbb{R}[X]$, then the linear operator $g(T)$ is zero

B. Suppose 0 is an eigenvalue of T with at least two linearly independent eigenvectors. If we write $f(X) = Xg(X)$ in $\mathbb{R}[X]$, then the linear operator $g(T)$ is zero.

Which of the following is true?

(a.) Both (A) and (B) are true

(b.) Both (A) and (B) are false

(c.) (A) is true and (B) is false

(d.) (A) is false and (B) is true

(39.) Let C be the positively oriented circle in the complex plane of radius 3 centered at the origin. What is the value of the integral $\int_C \frac{dz}{z^2(e^z - e^{-z})}$?

(a.) $i\pi/12$

(b.) $-i\pi/12$

(c.) $i\pi/6$

(d.) $-i\pi/6$

(40.) Let $f(z)=\exp\left(z+\frac{1}{z}\right), z\in\mathbb{C}\setminus\{0\}$. Then residue of f at $z=0$ is

(a.) $\sum_{l=0}^{\infty}\frac{1}{(l+1)!}$

(b.) $\sum_{l=0}^{\infty}\frac{1}{l!(l+1)}$

(c.) $\sum_{l=0}^{\infty}\frac{1}{l!(l+1)!}$

(d.) $\sum_{l=0}^{\infty}\frac{1}{(l^2+l)!}$

(41.) Suppose $\mathbf{X}=(X_1,X_2,X_3,X_4)^T$ has a multivariate normal $N_4(\mathbf{0}, I_2\otimes\Sigma)$, where I_2 is the 2×2 identity matrix, $\otimes$ is the Kronecker product, and $\sum=\begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}$. Define $Z=\begin{pmatrix}X_1 & X_2\\ X_3 & X_4\end{pmatrix}$ and $Q=((Q_{ij}))=Z^TZ$. Suppose χ_n^2 denotes a chi-square random variable with n degrees of freedom, and $W_m(n,\Sigma)$ denotes a Wishart distribution of order m with parameters n and Σ. The distribution of $(Q_{11}+Q_{12}+Q_{21}+Q_{22})$ is

(a.) $W_1(2,2)$

(b.) $W_1(1,2)$

(c.) $W_1(2,1)$

(d.) $2\chi_4^2$

(42.) Let X be a Poisson random variable with mean λ. Which of the following parametric function is not estimable?

(a.) λ^{-1}

(b.) λ

(c.) λ^2

(d.) $e^{-\lambda}$

(43.) Let $u(x,t)$ be the solution of

$u_{tt}-u_{xx}=0, 0<x<2, t>0,$

$u(0,t)=0=u(2,t), \forall t>0,$

$u(x,0)=\sin(\pi x)+2\sin(2\pi x), 0\le x\le 2,$

$u_t(x,0)=0,\ \ 0\le x\le 2.$

Which of the following is true?

(a.) $u(1,1)=-1$

(b.) $u(1/2, 1) = 0$

(c.) $u(1/2, 2) = 1$

(d.) $u_t(1/2, 1/2) = \pi$

(44.) Which of the following assertions is correct?

(a.) $\limsup_n e^{\cos\left(\frac{n\pi + (-1)^n 2e}{2n}\right)} > 1$

(b.) $\lim_n e^{\log_e\left(\frac{n\pi^2 + (-1)^n e^2}{7n}\right)}$ does not exist

(c.) $\liminf_n e^{\sin\left(\frac{n\pi + (-1)^n 2e}{2n}\right)} < \pi$

(d.) $\lim_n e^{\tan\left(\frac{n\pi^2 + (-1)^n e^2}{7n}\right)}$ does not exist

(45.) How many real roots does the polynomial $x^3 + 3x - 2023$ have?

(a.) 0
(b.) 1
(c.) 2
(d.) 3

(46.) Suppose S is an infinite set. Assuming that the axiom of choice holds, which of the following is true?

(a.) S is in bijection with the set of rational numbers
(b.) S is in bijection with the set of real numbers
(c.) S is in bijection with $S \times S$
(d.) S is in bijection with the power set of S

(47.) Let $X = (X_1, X_2)^T$ follow a bivariate normal distribution with mean vector $(0, 0)^T$ and covariance matrix Σ such that $\Sigma = \begin{bmatrix} 5 & -3 \\ -3 & 10 \end{bmatrix}$. The mean vector and covariance matrix of $Y = (X_1, 5 - 2X_2)^T$ are

(a.) $\begin{pmatrix} 0 \\ 5 \end{pmatrix}, \begin{bmatrix} 5 & -3 \\ -3 & 40 \end{bmatrix}$

(b.) $\begin{pmatrix} 0 \\ 5 \end{pmatrix}, \begin{bmatrix} 5 & -6 \\ -6 & 20 \end{bmatrix}$

(c.) $\begin{pmatrix} 0 \\ 5 \end{pmatrix}, \begin{bmatrix} 5 & 3 \\ 3 & 20 \end{bmatrix}$

(d.) $\begin{pmatrix} 0 \\ 5 \end{pmatrix}, \begin{bmatrix} 5 & 6 \\ 6 & 40 \end{bmatrix}$

(48.) Consider the random sample $\{3,6,9\}$ of size 3 from a normal distribution with mean $\mu \in (-\infty, 5]$ and variance 1. Then the maximum likelihood estimate of μ is

(a.) 6
(b.) 5
(c.) 3
(d.) 9

(49.) Suppose $X_1, X_2, \ldots, X_n$ are independently and identically distributed $N(\theta, 1)$ random variables, for $\theta \in \mathbb{R}$. Suppose $\bar{X} = n^{-1}\sum_{i=1}^{n} X_i$ denotes the sample mean and let $t_{0.975,n-1}$ denote the 0.975-quantile of a Student's-t distribution with $n-1$ degrees of freedom. Which of the following statements is true for the following interval $\bar{X} \pm t_{.975,\, n-1}\frac{1}{\sqrt{n}}$?

(a.) The interval is a confidence interval for θ with confidence level of exactly 0.95
(b.) The interval is a confidence interval for θ with confidence level being less than 0.95
(c.) The interval is a confidence interval for θ with confidence level being more than 0.95
(d.) The interval is not a confidence interval

(50.) The number of solutions of equation $x^2 = 1$ in the ring $\mathbb{Z}/105\mathbb{Z}$ is

(a.) 0
(b.) 2
(c.) 4
(d.) 8

(51.) Consider $\mathbb{R}$ with the usual topology. Which of the following assertions is correct?

(a.) A finite set containing 33 elements has at least 3 different Hausdorff topologies
(b.) Let X be a non-empty finite set with a Hausdorff topology. Consider $X \times X$ with the product topology. Then, every function $f : X \times X \to \mathbb{R}$ is continuous
(c.) Let X be a discrete topological space having infinitely many elements. Let $f : \mathbb{R} \to X$ be a continuous function and $g : X \to \mathbb{R}$ be any non-constant function. Then the range of $g \circ f$ contains at least 2 elements
(d.) If a non-empty metric space X has a finite dense subset, then there exists a discontinuous function $f : X \to \mathbb{R}$

(52.) Consider the simple linear regression model $Y_i = \beta x_i + \varepsilon_i$, for $i = 1, \ldots, n$; where $E(\varepsilon_i) = 0$, $Cov(\varepsilon_i, \varepsilon_k) = 0$ if $i \neq k$ and $Var(\varepsilon_i) = x_i^2\sigma^2$. The best linear unbiased estimator of β is:

(a.) $\dfrac{\sum_{i=1}^{n} Y_i x_i}{\sum_{i=1}^{n} x_i^2}$

(b.) $\dfrac{\sum_{i=1}^{n} Yx}{\sum_{i=1}^{n} x_i}$

(c.) $\dfrac{1}{n}\sum_{i=1}^{n}\dfrac{Y_i}{x_i}$

(d.) $\dfrac{1}{n}\sum_{i=1}^{n}\dfrac{Y_i x_i}{x_i^2}$

(53.) Let A be a 3×3 matrix with real entries. Which of the following assertions is FALSE?

(a.) A must have a real eigenvalue

(b.) If the determinant of A is 0, then 0 is an eigenvalue of A

(c.) If the determinant of A is negative and 3 is an eigenvalue of A, then A must have three real eigenvalues

(d.) If the determinant of A is positive and 3 is an eigenvalue of A, then A must have three real eigenvalues

(54.) Consider the variational problem (P)

$$J(y(x))=\int_0^1\left[(y')^2-y|y|y'+xy\right]dx,\ \ y(0)=0, y(1)=0.$$

Which of the following statements is correct?

(a.) (P) has no stationary function (extremal)

(b.) $y\equiv 0$ is the only stationary function (extremal) for (P)

(c.) (P) has a unique stationary function (extremal) y not identically equal to 0

(d.) (P) has infinitely any stationary functions (extremal)

(55.) Consider the constants a and b such that the following generalized coordinate transformation from (p,q) to (P,Q) is canonical $Q=pq^{(a+1)}, P=q^b$. What are the value of a and b?

(a.) $a=-1, b=0$

(b.) $a=-1, b=1$

(c.) $a=1, b=0$

(d.) $a=1, b=-1$

(56.) Which one of the following functions is uniformly continuous on the interval $(0,1)$?

(a.) $f(x)=\sin\dfrac{1}{x}$

(b.) $f(x)=e^{-1/x^2}$

(c.) $f(x)=e^x\cos\dfrac{1}{x}$

(d.) $f(x)=\cos x \cos\frac{\pi}{x}$

(57.) Which of the following equations can occur as the class equation of a group of order 10?

(a.) $10 = 1+1+\ldots+1\,(10-\text{times})$

(b.) $10 = 1+1+2+2+2+2$

(c.) $10 = 1+1+1+2+5$

(d.) $10 = 1+2+3+4$

(58.) Which of the following values of a,b,c and d will produce a quadrature formula $\int_{-1}^{1} f(x)\,dx \approx af(-1)+bf(1)+cf'(-1)+df'(1)$ that has degree precision 3?

(a.) $a=1, b=1, c=\frac{1}{3}, d=-\frac{1}{3}$

(b.) $a=-1, b=1, c=\frac{1}{3}, d=-\frac{1}{3}$

(c.) $a=1, b=1, c=-\frac{1}{3}, d=\frac{1}{3}$

(d.) $a=1, b=-1, c=\frac{1}{3}, d=-\frac{1}{3}$

(59.) Let $x=(x_1,\ldots,x_n)$ and $y=(y_1,\ldots,y_n)$ denote vectors in $\mathbb{R}^n$ for a fixed $n\ge 2$. Which of the following defines an inner product on $\mathbb{R}^n$?

(a.) $\langle x,y\rangle = \sum_{i,j=1}^{n} x_i y_j$

(b.) $\langle x,y\rangle = \sum_{i,j=1}^{n} \left(x_i^2 + y_j^2\right)$

(c.) $\langle x,y\rangle = \sum_{j=1}^{n} j^3 x_j y_j$

(d.) $\langle x,y\rangle = \sum_{j=1}^{n} x_j y_{n-j+1}$

(60.) Let p be a prime number. Let G be a group such that for each $g \in G$ there exists an $n \in \mathbb{N}$ such that $g^{p^n}=1$. Which of the following statements is FALSE?

(a.) If $|G| = p^6$, then G has a subgroup of index p^2

(b.) If $|G| = p^6$, then G has at least five normal subgroups

(c.) Center of G can be infinite

(d.) There exists G with $|G| = p^6$ such that G has exactly six normal subgroups

PART-C

(Mathematical Sciences)

(61.) Let $u = u(x,y)$ be the solution to the following Cauchy problem $u_x + u_y = e^u$ for $(x,y) \in \mathbb{R} \times \left(0, \frac{1}{e}\right)$ and $u(x,0) = 1$ for $x \in \mathbb{R}$. Which of the following statements are true?

(a.) $u\left(\frac{1}{2e}, \frac{1}{2e}\right) = 1$

(b.) $u_x\left(\frac{1}{2e}, \frac{1}{2e}\right) = 0$

(c.) $u_y\left(\frac{1}{4e}, \frac{1}{4e}\right) = \log 4$

(d.) $u_y\left(0, \frac{1}{4e}\right) = \frac{4e}{3}$

(62.) Consider the multiple linear regression model $Y = X\beta + \varepsilon$; where Y is $n \times 1$ observed data vector with $n > 5$; X is $n \times 5$ matrix of known constants with $rank(X) = 5$; $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)^T$ and $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)^T$, where ε_i for $i = 1, \ldots, n$ are independent and identically distributed $N(0,1)$ random variables. Consider testing of the linear hypothesis $H_0 = \beta_1 = \beta_2 = \beta_3 = \beta_4 = c$, (a known constant) against the alternative $H_1 : H_0$ is not true. Which of the following statements are true?

(a.) The sum of squares residuals under H_0 follows a central χ^2 distribution with $(n-5)$ degrees of freedom

(b.) The sum of squares residuals under H_0 follows a central χ^2 distribution with $(n-1)$ degrees of freedom

(c.) The test statistic follows a central F distribution with $(5, n-1)$ degrees of freedom

(d.) The test statistic follows a central F distribution with $(4, n-5)$ degrees of freedom

(63.) Let $f \in C^1(\mathbb{R})$ be bounded. Let us consider the initial-value problem

$$(P) \begin{cases} x'(t) = f(x(t)), t > 0, \\ x(0) = 0. \end{cases}$$

Which of the following statements are true?

(a.) (P) has solutions(s) defined for all $t > 0$

(b.) (P) has a unique solution

(c.) (P) has infinitely many solutions

(d.) The solution(s) of (P) is/are Lipschitz

(64.) Let $f : \{z : |z| < 1\} \to \left\{z : |z| \le \frac{1}{2}\right\}$ be a holomorphic function such that $f(0) = 0$. Which of the following statements are true?

(a.) $|f(z)| \le |z|$ for all z in $\{z : |z| < 1\}$

(b.) $|f(z)| \le \left|\frac{z}{2}\right|$ for all z in $\{z : |z| < 1\}$

(c.) $|f(z)| \le \frac{1}{2}$ for all z in $\{z : |z| < 1\}$

(d.) It is possible that $f\left(\frac{1}{2}\right) = \frac{1}{2}$

(65.) Let $n \ge 2$ be a positive integer. Consider a Markov chain on the state space $\{1, 2, \dots, n\}$ with a given transition probability matrix P. Let I_n denote the identity matrix of order n. Which of the following statements are necessarily true?

(a.) At least one state is recurrent

(b.) At least one state is transient

(c.) $-\frac{1}{3}I_n + \frac{4}{3}P$ is also a transition probability matrix of some Markov chain

(d.) 5 in an eigenvalue of $I_n + 3P + P^2$

(66.) Let V be the real vector space of a real polynomials in one variable with degree less than or equal to 10 (including the zero polynomial). Let $T : V \to V$ be the linear map defined by $T(p) = p'$, where p' denotes the derivative of p. Which of the following statements are correct?

(a.) $rank(T) = 10$

(b.) $\text{determinant}(T) = 0$

(c.) $trace(T) = 0$

(d.) All the eigenvalues of T are equal to 0

(67.) A cumulative hazard function $H(t)$ of a non-negative continuous random variable satisfies which of the following conditions?

(a.) $\lim_{t \to \infty} H(t) = \infty$

(b.) $H(0) = 0$

(c.) $H(1) = 1$

(d.) $H(t)$ is a non-decreasing function of t

(68.) Define a function $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = \begin{cases} \sin(\pi / x) & \text{when } x \ne 0, \\ 0 & \text{when } x = 0. \end{cases}$

On which of the following subsets of $\mathbb{R}$, the restriction of f is a continuous function?

(a.) $[-1, 1]$

(b.) $(0, 1)$

(c.) $\{0\} \cup \left\{\left(\frac{1}{n}\right) : n \in \mathbb{N}\right\}$

(d.) $\left\{\frac{1}{2^n} : n \in \mathbb{N}\right\}$

(69.) A point particle having unit mass is moving in x, y plane having the Lagrangian as follows $L = \dot{x}\dot{y} - 2x^2 - 2y^2$. What are possible values of p_r (conjugate momentum to radial coordinate in plane polar coordinate)?

(a.) $\dot{r}$

(b.) $\dot{r}\sin 2\theta + 2\dot{\theta}\cos 2\theta$

(c.) $\dot{r}\sin\theta + r\dot{\theta}\cos\theta$

(d.) $2\dot{r}\sin\theta + r\dot{\theta}\cos\theta$

(70.) Which of the following statements are correct?

(a.) The set of open right half-planes is a basis for the usual (Euclidean) topology on $\mathbb{R}^2$

(b.) The set of lines parallel to Y-axis is a basis for the dictionary order topology on $\mathbb{R}^2$

(c.) The set of open rectangles is a basis for the usual (Euclidean) topology on $\mathbb{R}^2$

(d.) The set of line segments (without end points) parallel to Y-axis is a basis for the dictionary order topology on $\mathbb{R}^2$

(71.) Let A, B be two events in a discrete probability space with $\mathbb{P}(A) > 0$ and $\mathbb{P}(B) > 0$. Which of the following are necessarily true?

(a.) If $\mathbb{P}(A|B) = 0$ then $\mathbb{P}(B|A) = 0$

(b.) If $\mathbb{P}(A|B) = 1$ then $\mathbb{P}(B|A) = 1$

(c.) If $\mathbb{P}(A|B) > \mathbb{P}(A)$ then $\mathbb{P}(B|A) > \mathbb{P}(B)$

(d.) If $\mathbb{P}(A|B) > \mathbb{P}(B)$ then $\mathbb{P}(B|A) > \mathbb{P}(A)$

(72.) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = x^2 - y^3$. Which of the following statements are true?

(a.) There is no continuous real-valued function g defined on any interval of $\mathbb{R}$ containing 0 such that $f(x, g(x)) = 0$

(b.) There is exactly one continuous real-valued function g defined on an interval of $\mathbb{R}$ containing 0 such that $f(x, g(x)) = 0$

(c.) There is exactly one differentiable real-valued function g defined on an interval of $\mathbb{R}$ containing 0 such that $f(x, g(x)) = 0$

(d.) There are two distinct differentiable real-valued functions g on an interval of $\mathbb{R}$ containing 0 such that $f(x, g(x)) = 0$

(73.) Let X and Y be independent Poisson random variables with parameters 2 and 3, respectively. Which of the following statements are correct?

(a.) $Var(X | X + Y = 2) = \frac{12}{25}$

(b.) $E\left(\frac{2}{1+X} \mid X+Y=2\right)=\frac{98}{3}$

(c.) $P\left(X^2=0 \mid X+Y=2\right)=e^{-2}+\frac{9}{25}\left(1-e^{-2}\right)$

(d.) $X \mid Y=3 \sim Binomial(3,2)$

(74.) Let E be a finite algebraic Galois extension of F with Galois group G. Which of the following statements are true?

(a.) There is an intermediate field K with $K \neq F$ and $K \neq E$ such that K is a Galois extension of F

(b.) If every proper intermediate field K is a Galois extension of F then G is Abelian

(c.) If E has exactly three intermediate field including F and E then G is Abelian

(d.) If $[E:F]=99$ then every intermediate field is a Galois extension of F

(75.) Let X_i, for $i=1,2,\ldots,2n$, $n \geq 1$, be independent random variables each distributed as $N(0,1)$. Which of the following statements are correct?

(a.) $\left(X_1+\ldots+X_n-X_{n+1}-\ldots-X_{2n}\right)/2n \sim N(0,2)$

(b.) $\left(X_1-X_2\right)^2+\left(X_3-X_4\right)^2+\ldots+\left(X_{2n-1}-X_{2n}\right)^2 \sim 2\chi_n^2$

(c.) $E\left[\max\left(\left|X_1\right|,\left|X_{n+1}\right|\right)\right]=\frac{2}{\sqrt{\pi}}$

(d.) $E\left[\max\left(\left|X_1\right|,\left|X_{n+1}\right|\right)\right]=\frac{4}{\sqrt{\pi}}$

(76.) Let $K \in C([0,1]\times[0,1])$ satisfy $|K(x,y)|<1$ for all $x,y \in [0,1]$. For every $g \in C[0,1]$, let us consider the integral equation (P_g) $f(x)+\int_0^1 K(x,y) f(y)\,dy = g(x)$, for all $x \in [0,1]$. Which of the following statements are true?

(a.) There exists a $g \in C[0,1]$ for which (P_g) has no solution in $C[0,1]$

(b.) (P_g) has a solution in $C[0,1]$ for infinitely many $g \in C[0,1]$

(c.) The solution of (P_g) in $C[0,1]$ is unique if $g \in C^1[0,1]$

(d.) There exists a $g \in C[0,1]$ for which (P_g) has infinitely many solutions in $C[0,1]$

(77.) Let $D=\{z \in \mathbb{C} : |z|<1\}$. Consider the following statements

A. $f: D \to D$ be a holomorphic function. Suppose α, β are distinct complex numbers in D such that $f(\alpha)=\alpha$ and $f(\beta)=\beta$. Then $f(z)=z$ for all $z \in D$

B. There does not exist a bijective holomorphic function from D to the set of all complex numbers whose imaginary part is positive

C. $f: D \to D$ be a holomorphic function. Suppose $\alpha \in D$ be such that $f(\alpha)=\alpha$ and $f'(\alpha)=1$. Then $f(z)=z$ for all $z \in D$.

Which of the following options are true?

(a.) (A), (B) and (C) are true
(b.) (A) is true
(c.) Both (A) and (B) are false
(d.) Both (A) and (C) are true

(78.) Let $f(z)$ be an entire function on $\mathbb{C}$. Which of the following statements are true?

(a.) $f(\bar{z})$ is an entire function

(b.) $\overline{f(z)}$ is an entire function

(c.) $\overline{f(\bar{z})}$ is an entire function

(d.) $\overline{f(z)}+f(\bar{z})$ is an entire function

(79.) Consider the following two sequences $\{a_n\}$ and $\{b_n\}$ given by

$a_n = \frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}$, $b_n = \frac{1}{n}$.

Which of the following statements are true?

(a.) $\{a_n\}$ converges to $\log 2$, and has the same convergence rate as the sequence $\{b_n\}$

(b.) $\{a_n\}$ converges to $\log 4$, and has the same convergence rate as the sequence $\{b_n\}$

(c.) $\{a_n\}$ converges to $\log 2$, but does not have the same convergence rate as the sequence $\{b_n\}$

(d.) $\{a_n\}$ does not converge

(80.) Let V be the vector space of all polynomials in one variable of degree at most 10 with real coefficients. Let W_1 be the subspace of V consisting of polynomials of degree at most 5 and let W_2 be the subspace of V consisting of polynomials such that the sum of their coefficients is 0. Let W be the smallest subspace of V containing both W_1 and W_2. Which of the following statements are true?

(a.) The dimension of W is at most 10

(b.) $W = V$

(c.) $W_1 \subset W_2$

(d.) The dimension of $W_1 \cap W_2$ is at most 5

(81.) Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc and C the positively oriented boundary $\{|z| = 1\}$. Fix a finite set $\{z_1, z_2, \ldots, z_n\} \subseteq \mathbb{D}$ of distinct points and consider the polynomial $g(z) = (z-z_1)(z-z_2)\ldots(z-z_n)$ of degree n. Let f be a holomorphic function in an open neighbourhood of $\bar{\mathbb{D}}$ and define $P(z) = \frac{1}{2\pi i}\int_C f(\zeta)\frac{g(\zeta)-g(z)}{(\zeta-z)g(\zeta)}d\zeta$.

(a.) P is a polynomial of degree n

(b.) P is a polynomial of degree $n-1$

(c.) P is a rational function on $\mathbb{C}$ with poles at $z_1, z_2, \ldots, z_n$

(d.) $P(z_j) = f(z_j)$ for $j = 1, 2, \ldots, n$

(82.) Let $u : \mathbb{R}^2 \to \mathbb{R}$ be the solution to the Cauchy problem:

$$\begin{cases} \partial_x u + 2\partial_y u = 0 & \text{for } (x, y) \in \mathbb{R}^2, \\ u(x, y) = \sin(x) & \text{for } y = 3x + 1, \ x \in \mathbb{R}. \end{cases}$$

Let $v : \mathbb{R}^2 \to \mathbb{R}$ be the solution to the Cauchy problem:

$$\begin{cases} \partial_x v + 2\partial_y v = 0 & \text{for } (x, y) \in \mathbb{R}^2, \\ v(x, 0) = \sin(x) & \text{for } x \in \mathbb{R}. \end{cases}$$

Let $\mathcal{S} = [0, 1] \times [0, 1]$.

Which of the following statements are true?

(a.) u changes sign in the interior of $\mathcal{S}$

(b.) $u(x, y) = v(x, y)$ along a line in $\mathcal{S}$

(c.) v changes sign in the interior of $\mathcal{S}$

(d.) v vanishes along a line in $\mathcal{S}$

(83.) Let $\{X_i : 1 \le i \le 2n\}$ be independently and identically distributed normal random variables with mean μ and variance 1, and independent of a standard Cauchy random variable W . Which of the following statistics are consistent for μ?

(a.) $n^{-1} \sum_{i=1}^{n} X_i$

(b.) $n^{-1} \sum_{i=1}^{2n} X_i$

(c.) $n^{-1} \sum_{i=1}^{n} X_{2i-1}$

(d.) $n^{-1} \left(\sum_{i=1}^{n} X_i + W \right)$

(84.) Define $f : \mathbb{R}^4 \to \mathbb{R}$ be $f(x, y, z, w) = xw - yz$. Which of the following statements are true?

(a.) f is continuous

(b.) If $U = \{(x, y, z, w) \in \mathbb{R}^4 : xy + zw = 0, \ x^2 + z^2 = 1, \ y^2 + w^2 = 1\}$, then f is uniformly continuous on U

(c.) If $V = \{(x, y, z, w) \in \mathbb{R}^4 : x = y, \ z = w\}$, then f is uniformly continuous on V

(d.) If $W = \{(x, y, z, w) \in \mathbb{R}^4 : 0 \le x + y + z + w \le 1\}$, then f is unbounded on W

(85.) Under which of the following conditions is the sequence $\{x_n\}$ of real numbers convergent?

(a.) The subsequences $\{x_{(2n+1)}\}, \{x_{2n}\}$ and $\{x_{3n}\}$ are convergent and have the same limit

(b.) The subsequences $\{x_{(2n+1)}\}, \{x_{2n}\}$ and $\{x_{3n}\}$ are convergent (c.) The subsequences $\{x_{kn}\}_n$ are convergent for every $k \geq 2$

(d.) $\lim_n |x_{(n+1)} - x_n| = 0$

(86.) Suppose under the null hypothesis H, $X \sim p$, where $p(x) = P(X = x) = 1/20$, $x \in \{1, 2, \ldots, 20\}$ and under the alternative hypothesis K, $X \sim q$ where $q(x) = P(X = x) = \frac{x}{210}$, $x \in \{1, 2, \ldots, 20\}$. Define two test functions ϕ and ψ for testing H against K such that, $\phi(x) = \begin{cases} 1 & \text{if } x \leq 2, \\ 0 & \text{otherwise,} \end{cases}$ and $\psi(x) = \begin{cases} 1 & \text{if } x \geq 19, \\ 0 & \text{otherwise.} \end{cases}$. Which of the following statements are true?

(a.) Size of the test ϕ is 0.1

(b.) Size of the test ψ is 0.05

(c.) (Power of the test ψ) > 0.05

(d.) (Power of the test ψ) > (Power of the test ϕ)

(87.) Let $X = \prod_{n=1}^{\infty} [0, 1]$, that is, the space of the sequences $\{x_n\}_{n \geq 1}$ with $x_n \in [0, 1]$, $n \geq 1$. Define the metric $d : X \times X \to [0, \infty)$ by $d(\{x_n\}_{n \geq 1}, \{y_n\}_{n \geq 1}) = \sup_{n \geq 1} \frac{|x_n - y_n|}{2^n}$. Which of the following statements are true?

(a.) The metric topology on X is finer than the product topology on X

(b.) The metric topology on X is coarser then the product topology on X

(c.) The metric topology on X is same as the product topology on X

(d.) The metric topology on X and the product topology on X are not comparable

(88.) Suppose $X_1, X_2, \ldots, X_n$ are independently and identically distributed $N(0, \tau^{-2})$ random variables, for $\tau^{-2} > 0$. Let the prior distribution on τ^2 have density $\pi(\tau^2) \propto \left(\frac{1}{\tau^2}\right)^{\alpha}$ for some $\alpha > 0$. Which of the following are true?

(a.) The prior distribution is an improper distribution for $\alpha > 0$

(b.) The posterior distribution is a proper distribution for all $\alpha > 0$

(c.) Under a squared error loss, the generalized Bayes estimator of τ^2 is $\frac{n/2 - \alpha}{\sum_{i=1}^{n} X_i^2 / 2}$

(d.) The posterior distribution is proper for $\alpha = 1$

(89.) Suppose A is a 5×5 block diagonal real matrix with diagonal blocks $\begin{pmatrix} e & 1 \\ 0 & e \end{pmatrix}, \begin{pmatrix} e & 1 & 0 \\ 0 & e & 0 \\ 0 & 0 & e \end{pmatrix}$. Which of the following statements are true?

(a.) The algebraic multiplicity of e in A is 5

(b.) A is not diagonalisable

(c.) The geometric multiplicity of e in A is 3
(d.) The geometric multiplicity of e in A is 2

(90.) Suppose that $\mathbf{X}_1,\ldots,\mathbf{X}_n,\mathbf{X}_{n+1}$ is a random sample of size $\mathbf{n}+1$, where $p>2$ and $\mathbf{n}>p+3$, from a multivariate normal population, $N_p(\boldsymbol{\mu},\boldsymbol{\Sigma})$; $\boldsymbol{\mu}\in\mathcal{R}^p$ and $\boldsymbol{\Sigma}>0$. Let $\bar{\mathbf{X}}_n=\frac{1}{n}\sum_{i=1}^{n}\mathbf{X}_i$ and $(\mathbf{n}-\mathbf{1})\mathbf{S}$ $=\sum_{i=1}^{n}(\mathbf{X}_i-\bar{\mathbf{X}}_n)(\mathbf{X}_i-\bar{\mathbf{X}}_n)^{\mathbf{T}}$. Which of the following are correct?

(a.) $(\bar{\mathbf{X}}_n-\mathbf{X}_{n+1})^{\mathbf{T}}\mathbf{S}^{-1}(\bar{\mathbf{X}}_n-\mathbf{X}_{n+1})\sim\frac{\mathbf{p}(\mathbf{n}^2-1)}{\mathbf{n}(\mathbf{n}-\mathbf{p})}\mathbf{F}_{\mathbf{p},\,\mathbf{n}-\mathbf{p}}$

(b.) $E(\bar{\mathbf{X}}_n^{\mathbf{T}}\mathbf{S}\bar{\mathbf{X}}_n)=trace\left(\frac{\boldsymbol{\Sigma}^2}{\mathbf{n}}\right)+\boldsymbol{\mu}^{\mathbf{T}}\boldsymbol{\Sigma}\boldsymbol{\mu}$

(c.) $E(\mathbf{S}^{-1})=\frac{\mathbf{n}-\mathbf{1}}{\mathbf{n}-\mathbf{p}-\mathbf{2}}\sum^{-1}$

(d.) $(\bar{\mathbf{X}}_n-\mathbf{X}_{n+1})^{T}\sum^{-1}(\bar{\mathbf{X}}_n-\mathbf{X}_{n+1})\sim\frac{\mathbf{n}+\mathbf{1}}{\mathbf{n}}\chi_p^2$

(91.) Which of the following statements are true for an arbitrary normed linear space U ?
(a.) Every bounded linear functional from U to $\mathbb{R}$ is continuous
(b.) U is isomorphic to its double-dual U^{**}
(c.) For every $x\in U$, we have $\|x\|=\lim_{\|f\|\le 1}|f(x)|$, where f denotes elements of U^*
(d.) The closed unit ball in U is compact

(92.) Let $\lambda_1<\lambda_2$ be two real characteristic numbers for the following homogenous integral equation: $\varphi(x)=\lambda\int_0^{2\pi}\sin(x+t)\,\varphi(t)\,dt$; and let $\mu_1<\mu_2$ be two real characteristic numbers for the following homogenous integral equation: $\psi(x)=\mu\int_0^{\pi}\cos(x+t)\psi(t)\,dt$. Which of the following statements are true?
(a.) $\mu_1<\lambda_1<\lambda_2<\mu_2$
(b.) $\lambda_1<\mu_1<\mu_2<\lambda_2$
(c.) $|\mu_1-\lambda_1|=|\mu_2-\lambda_2|$
(d.) $|\mu_1-\lambda_1|=2|\mu_2-\lambda_2|$

(93.) Which of the following statements are correct?
(a.) If G is a group of order 244, then G contains a unique subgroup of order 27
(b.) If G is a group of order 1694, then G contains a unique subgroup of order 121
(c.) There exists a group of order 154 which contains a unique subgroup of order 7
(d.) There exists a group of order 121 which contains two subgroups of order 11

(94.) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation satisfying $T^3 - 3T^2 = -2I$, where $I:\mathbb{R}^3 \to \mathbb{R}^3$ is the identity transformation. Which of the following statements are true?

(a.) $\mathbb{R}^3$ must admit a basis $\mathcal{B}_1$ such that the matrix of T with respect to $\mathcal{B}_1$ is symmetric

(b.) $\mathbb{R}^3$ must admit a basis $\mathcal{B}_2$ such that the matrix of T with respect to $\mathcal{B}_2$ is upper triangular

(c.) $\mathbb{R}^3$ must contains a non-zero vector v such that $Tv = v$

(d.) $\mathbb{R}^3$ must contain two linearly independent vectors v_1, v_2 such that $Tv_1 = v_1$ and $Tv_2 = v_2$

(95.) Let a continuous random variable X follow $Uniform(-1,1)$. Define $Y = X^2$. Which of the following are NOT true for X and Y?

(a.) They are independent and uncorrelated

(b.) They are independent but correlated

(c.) They are not independent but correlated

(d.) They are neither independent nor correlated

(96.) Suppose $y(x)$ is an extremal of the following functional

$J(y(x)) = \int_0^1 \left(y(x)^2 - 4y(x)y'(x) + 4y'(x)^2\right)dx$ subject to $y(0) = 1$ and $y'(0) = \frac{1}{2}$. Which of the following statements are true?

(a.) y is a convex function

(b.) y is a concave function

(c.) $y(x_1 + x_2) = y(x_1)y(x_2)$ for all x_1, x_2 in $[0,1]$

(d.) $y(x_1 x_2) = y(x_1) + y(x_2)$ for all $x_1, x_2\,[0,1]$

(97.) Consider the following initial value problem (IVP), $\frac{du}{dt} = t^2 u^{1/5}, u(0) = 0$. Which of the following statements are correct?

(a.) The function $g(t,u) = t^2 u^{1/5}$ does not satisfy the Lipschitz's condition with respect to u in any neighbourhood of $u = 0$

(b.) There is no solution for the IVP

(c.) There exist more than one solution for the IVP

(d.) The function $g(t,u) = t^2 u^{1/5}$ satisfies the Lipschitz's condition with respect to u in some neighbourhood of $u = 0$ and hence there exists a unique solution for the IVP

(98.) Suppose that $f:\mathbb{R}^n \to \mathbb{R}$, $n \ge 2$ is a C^2 function satisfying $f(y) \ge f(x) + \nabla(f)(x)(y-x)$ for every x, y in $\mathbb{R}^n$. Here ∇ denotes the gradient. Which of the following statements are true?

(a.) f is constant

(b.) f is convex

(c.) f is convex and bounded

(d.) f is constant if f is bounded

(99.) Consider the linear model $E(Y_1)=2\beta_1-\beta_2-\beta_3$, $E(Y_2)=\beta_2-\beta_4$, $E(Y_3)=\beta_2+\beta_3-2\beta_4$ with uncorrelated and homoscedastic random error. Which of the following linear parametric functions are estimable?

(a.) $16\beta_1-9\beta_2-7\beta_3$

(b.) $\beta_3-\beta_4$

(c.) $57\beta_1-18\beta_2-13\beta_3-26\beta_4$

(d.) $23\beta_1-9\beta_2-10\beta_3+4\beta_4$

(100.) Which of the following are maximal ideals of $\mathbb{Z}[X]$?

(a.) Ideal generated by 2 and $(1+X^2)$

(b.) Ideal generated by 2 and $(1+X+X^2)$

(c.) Ideal generated by 3 and $(1+X^2)$

(d.) Ideal generated by 3 and $(1+X+X^2)$

(101.) Let V be a finite dimensional real vector space and T_1,T_2 be two nilpotent operators on V. Let $W_1=\{v\in V:T_1(v)=0\}$ and $W_2=\{v\in V:T_2(v)=0\}$. Which of the following statements are FALSE?

(a.) If T_1 and T_2 are similar, then W_1 and W_2 are isomorphic vector spaces

(b.) If W_1 and W_2 are isomorphic vector spaces, then T_1 and T_2 have the same minimal polynomial

(c.) If $W_1=W_2=V$, then T_1 and T_2 are similar

(d.) If W_1 and W_2 are isomorphic, then T_1 and T_2 have the same characteristic polynomial

(102.) Let G be a group of order 2023. Which of the following statements are true?

(a.) G is an Abelian group

(b.) G is cyclic group

(c.) G is a simple group

(d.) G is not a simple group

(103.) Let $\{x_n\}$ be a sequence of positive real numbers. If $\sigma_n=\frac{1}{n}(x_1+x_2+\ldots x_n)$, then which of the following are true? (Here lim sup denotes the limit supremum of a sequence.)

(a.) If $\limsup\{x_n\}=\ell$ and $\{x_n\}$ is decreasing, then $\limsup\{\sigma_n\}=\ell$

(b.) $\limsup\{x_n\}=\ell$ if and only if $\limsup\{\sigma_n\}=\ell$

(c.) If $\limsup\left\{n\left(\frac{x_n}{x_{(n+1)}}-1\right)\right\}<1$, then $\sum_n x_n$ is convergent

(d.) If $\limsup\left\{n\left(\frac{x_n}{x_{(n+1)}}-1\right)\right\}<1$, then $\sum_n x_n$ is divergent

(104.) Let $(X_1,Y_1),\ldots,(X_4,Y_4)$ be a random sample from a continuous bivariate distribution function $F_{X,Y}$ with marginal distributions of X and Y being F_X and F_Y respectively. In order to test the null hypothesis H_0 :'X and Y are independent' against the alternative H_1 :'X and Y are positively associated', consider the Kendall sample correlation statistic $K=\sum_{i=1}^{3}\sum_{j=i+1}^{4}\psi\left((X_i,Y_i),(X_j,Y_j)\right)$, where $\psi\left((a,b),(c,d)\right)=\begin{cases}1, & \text{if } (d-b)(c-a)>0,\\ -1, & \text{if } (d-b)(c-a)<0.\end{cases}$. Assuming no ties, which of the following are true?

(a.) The test that rejects H_0 for $K \geq 4$ has size $1/4$

(b.) The test that rejects H_0 for $K \geq 4$ has size $1/6$

(c.) $P_{H_0}(K=4)=3/24$

(d.) $P_{H_0}(K=6)=1/12$

(105.) Let X_1 and X_2 be two independent random variables such that X_1 follows a gamma distribution with mean 10 variance 10, and $X_2 \sim N(3,4)$. Let f_1 and f_2 denote the density functions of X_1 and X_2, respectively. Define a new random variable Y so that for $y\in\mathbb{R}$, it has density function $f(y)=0.4f_1(y)+qf_2(y)$. Which of the following are true?

(a.) $q=0.6$

(b.) $E[Y]=5.8$

(c.) $Var(Y)=3.04$

(d.) $Y=0.4X_1+qX_2$

(106.) Suppose that cars arrive at a petrol pump following a Poisson distribution at the rate of 10 per hour. The time to perform the refilling is exponentially distributed and the single available staff takes an average of 4 minutes to refill each car. Further assume that the cards leave immediately after refilling. Let α denote the probability of finding 3 or more cars waiting to refill and let β denote the mean number of cars in the queue. Which of the following statements are correct?

(a.) $\alpha=\frac{8}{27}$

(b.) $\beta=1$

(c.) $\beta-\alpha=\frac{46}{27}$

(d.) $\alpha\beta=3$

(107.) Let us consider the following two initial value problems

$$(P)\begin{cases}x'(t)=\sqrt{x(t)}, t>0,\\ x(0)=0,\end{cases} \quad \text{and} \quad (Q)\begin{cases}y'(t)=-\sqrt{y(t)}, t>0,\\ y(0)=0.\end{cases}$$

Which of the following statements are true?

(a.) (P) has a unique solution in $[0,\infty)$

(b.) (Q) has a unique solution in $[0, \infty)$

(c.) (P) has infinitely many solution in $[0, \infty)$

(d.) (Q) has infinitely many solutions in $[0, \infty)$

(108.) Let μ denote the Lebesgue measure on $\mathbb{R}$ and μ^* be the associated Lebesgue outer measure. Let A be a non-empty subset of $[0,1]$. Which of the following statements are true?

(a.) If both interior and closure of A have the same outer measure, then A is Lebesgue measurable

(b.) If A is open, then A is Lebesgue measurable and $\mu(A) > 0$

(c.) If A is not Lebesgue measurable, then the set of irrationals in A must have positive outer measure

(d.) If $\mu^*(A) = 0$, then A has empty interior

(109.) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = \frac{1}{4} + x - x^2$. Given $a \in \mathbb{R}$, let us define the sequence $\{x_n\}$ by $x_0 = a$ and $x_n = f(x_{n-1})$ for $n \geq 1$. Which of the following statements are true?

(a.) If $a = 0$, then the sequence $\{x_n\}$ converges to $\frac{1}{2}$

(b.) If $a = 0$, then the sequence $\{x_n\}$ converges to $-\frac{1}{2}$

(c.) The sequence $\{x_n\}$ converges for every $a \in \left(-\frac{1}{2}, \frac{3}{2}\right)$, and it converges to $\frac{1}{2}$

(d.) If $a = 0$, then the sequence $\{x_n\}$ does not converge

(110.) Suppose $X_1, X_2, \ldots$ are independent and identically distributed $N(0,1)$ random variables and $Y_n = X_1^4 + X_2^2 + \ldots + X_n^4$. Which of the following probabilities converge to $\frac{1}{2}$ as $n \to \infty$?

(a.) $\mathbb{P}\{Y_n \in [0, 2n]\}$

(b.) $\mathbb{P}\{Y_n \in [n, 3n]\}$

(c.) $\mathbb{P}\{Y_n \in [2n, 4n]\}$

(d.) $\mathbb{P}\{Y_n \in [3n, 5n]\}$

(111.) Which of the following statements are true?

(a.) Maximum likelihood estimator may not be unique

(b.) A complete statistic is always sufficient

(c.) A sufficient statistic may not be complete

(d.) Any function of a sufficient statistic is always sufficient

(112.) Let $n \geq 1$ be a positive integer and S_n the symmetric group on n symbols.

Let $\Delta = \{(g,g) : g \in S_n\}$. Which of the following statements are necessarily true?

(a.) The map $f : S_n \times S_n \to S_n$ given by $f(a,b) = ab$ is a group homomorphism

(b.) Δ is a subgroup of $S_n \times S_n$

(c.) Δ is a normal subgroup of $S_n \times S_n$

(d.) Δ is a normal subgroup of $S_n \times S_n$, if n is a prime number

(113.) Let V be an inner product space and let $v_1, v_2, v_3 \in V$ be an orthogonal set of vectors. Which of the following statements are true?

(a.) The vectors $v_1 + v_2 + 2v_3$, $v_2 + v_3$, $v_2 + 3v_3$ can be extended to a basis of V

(b.) The vectors $v_1 + v_2 + 2v_3$, $v_2 + v_3$, $v_2 + 3v_3$ can be extended to an orthogonal basis of V

(c.) The vectors $v_1 + v_2 + 2v_3$, $v_2 + v_3$, $2v_1 + v_2 + 3v_3$ can be extended to a basis of V

(d.) The vectors $v_1 + v_2 + 2v_3$, $2v_1 + v_2 + v_3$, $2v_1 + v_2 + 3v_3$ can be extended to a basis of V

(114.) Which of the following are true?

(a.) For $n \geq 1$, the sequence of functions $f_n : (0,1) \to (0,1)$ defined by $f_n(x) = x^n$ is uniformly convergent

(b.) For $n \geq 1$, the sequence of functions $f_n : (0,1) \to (0,1)$ defined by $f_n(x) = \dfrac{x^n}{\log(n+1)}$ is uniformly convergent

(c.) For $n \geq 1$, the sequence of functions $f_n : (0,1) \to (0,1)$ defined by $f_n(x) = \dfrac{x^n}{1+x^n}$ is uniformly convergent

(d.) For $n \geq 1$, the sequence of functions $f_n : (0,1) \to (0,1)$ defined by $f_n(x) = \dfrac{x^n}{1+nx^n}$ is not uniformly convergent

(115.) Let G_1 and G_2 be two groups and $\varphi : G_1 \to G_2$ be a surjective group homomorphism. Which of the following statements are true?

(a.) If G_1 is cyclic then G_2 is cyclic

(b.) If G_1 is Abelian then G_2 is Abelian

(c.) If H is a subgroup of G_1 then $\varphi(H)$ is a subgroup of G_2

(d.) If N is a normal subgroup of G_1 then $\varphi(N)$ is a normal subgroup of G_2

(116.) Let $y(x)$ and $z(x)$ be the stationary function (extremals) of the variational problem $J(y(x), z(x)) = \int_0^1 \left[(y')^2 + (z')^2 + y'z'\right] dx$ subject to $y(0) = 1$, $y(1) = 0$, $z(0) = -1$, $z(1) = 2$. Which of the following statements are correct?

(a.) $z(x) + 3y(x) = 2$ for $x \in [0,1]$

(b.) $3z(x) + y(x) = 2$ for $x \in [0,1]$

(c.) $y(x)+z(x)=2x$ for $x\in[0,1]$

(d.) $y(x)+z(x)=x$ for $x\in[0,1]$

(117.) Consider the following quadratic forms over $\mathbb{R}$

A. $6X^2-13XY+6Y^2$

B. $X^2-XY+2Y^2$

C. $X^2-XY-2Y^2$

Which of the following statements are true?

(a.) Quadratic forms (A) and (B) are equivalent

(b.) Quadratic forms (A) and (C) are equivalent

(c.) Quadratic form (B) is positive definite

(d.) Quadratic form (C) is positive definite

(118.) Consider the following statements:

A. Let f be a continuous function on $[1,\infty)$ taking non-negative values such that $\int_1^\infty f(x)\,dx$ converges. Then $\sum_{n\geq 1} f(n)$ converges.

B. Let f be a function on $[1,\infty)$ taking non-negative values such that $\int_1^\infty f(x)\,dx$ converges. Then $\lim_{x\to\infty} f(x)=0$.

C. Let f be continuous, decreasing function on $[1,\infty)$ taking non-negative values such that $\int_1^\infty f(x)\,dx$ does not converge. Then $\sum_{n\geq 1} f(n)$ does not converge.

Which of the following options are true?

(a.) (A), (B) and (C) all are true

(b.) Both (A) and (B) are false

(c.) (C) is true

(d.) (D) is true

(119.) Let B be a 3×5 matrix with entries from $\mathbb{Q}$. Assume that $\{v\in\mathbb{R}^5 \mid Bv=0\}$ is a three-dimensional real vector space. Which of the following statements are true?

(a.) $\{v\in\mathbb{Q}^5 \mid Bv=0\}$ is a three-dimensional vector space over $\mathbb{Q}$

(b.) The linear transformation $T:\mathbb{Q}^3\to\mathbb{Q}^5$ given by $T(v)=B^t v$ is injective

(c.) The column span of B is two-dimensional

(d.) The linear transformation $T:\mathbb{Q}^3\to\mathbb{Q}^3$ given by $T(v)=BB^t v$ is injective

(120.) Let $Y_i=\alpha+\beta x_i+\varepsilon_i$, $i=1,2,3$, where x_i's are fixed covariates, α and β are unknown parameters and ε_i's are independently and identically distributed normal random variables with mean 0 and variance $\sigma^2 > 0$. Given the following observations,

x_i	1	2	3
y_i	2.1	3.9	6

which of the following statements are true?

(a.) Maximum likelihood estimate of α is 0.1

(b.) Least square estimate of α is 0.1

(c.) Best linear unbiased estimate of α is 0.1

(d.) Maximum likelihood estimate of $\frac{\beta}{\alpha}$ is 19.5